Mathematics as code

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April 4, 2024

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Foundation models in expert domains

LLMs in expert domains

- Finance
- Medicine
- Mathematics

[1] See, e.g. Harvard New Technologies in Mathematics: 1/2021 (Christian Szegedy), 3/2022 (Stan Polu), 10/2022 (Guy Gur-Ari), 10/2023 (Alex Gu), 12/2023 (Katherine Collins), ...;

Foundation models in expert domains

LLMs in expert domains

- Finance
- Medicine
- Mathematics
  - Solve problems
  - Open-ended dialogue
  - Help generate verified proofs
  - ...

[1] See, e.g. Harvard New Technologies in Mathematics: 1/2021 (Christian Szegedy), 3/2022 (Stan Polu), 10/2022 (Guy Gur-Ari), 10/2023 (Alex Gu), 12/2023 (Katherine Collins), ...;

"Informal" mathematics

Math as raw data (text, images, ...)

- Flexible
- Useful (e.g., in education)
- No correctness guarantees

Input:
Let r be a root of $x^2 + 5x + 7 = 0$. Compute $(r-1)(r+2)(r+6)(r+3)$.

Llemma 34B solution:
We have

$$\begin{aligned}(r-1)(r+2)(r+6)(r+3) &= (r^2+r-2)(r^2+9r+18)\\ &= r^4+10r^3+25r^2+19r-36\\ &= r^4+5r^3+7r^2+5r+7+5r^3+20r^2+14r-36\\ &= r^4+5r^3+25r^2+19r-29\\ &= (r^2+5r+7)(r^2+4r-5)\\ &= (0)(r^2+4r-5)\\ &= \boxed{0}.\end{aligned}$$

Final Answer: The final answer is 0.

Figure 1: Language model solution.

Formal mathematics

Math as **source code**

- Write a specification (e.g., 1+1=2)
- Write a proof
- **Automatically checked**
  - Code compiles ≡ correct proof

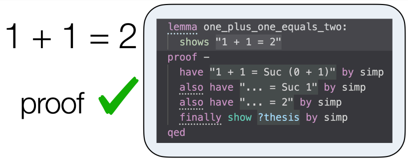

Figure 2: Math as source code.

Math as **source code**

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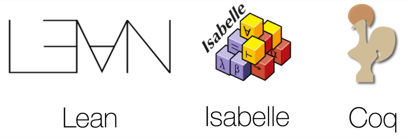

Figure 3: Theorem proving languages

If $R \subseteq S$ and $S \subseteq T$ then $R \subseteq T$

How is formal math used in practice?

Growing use in mathematics:

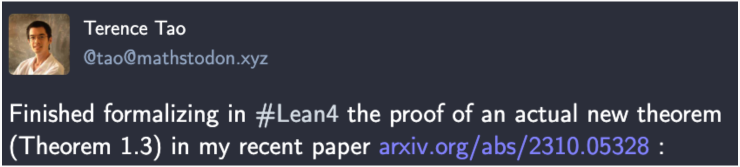

Figure 4: Terence Tao's Lean formalization project (October 2023)

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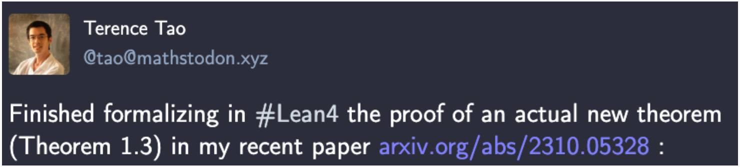

Figure 4: Terence Tao's Lean formalization project (October 2023)

- **Lean Mathlib** project: 1+ million lines of code, 300+ contributors
- **Courses** at CMU, Imperial College London, Fordham, JHU, ...
- **New journal (2024)**: *Annals of Formalized Mathematics*

How is formal math used in practice?

Why?[1]

- Collaboration
- Instant feedback
- Correctness guarantees
- ...

[1] See e.g., *Mathematics and the formal turn*, AFM Aims and Scope

Why is AI ∩ formal math important?

LLMs for formal math

- Automate proofs
- Translate informal to formal
- Suggest strategies
- …

Why is AI ∩ formal math important?

Formal math for LLMs

- Verifiable
  - Prevent incorrect math and code generation
  - Feedback signal for learning

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- Tests reasoning
  - From easy: 1+1 = 2
  - To hard: Fermat's Last Theorem

Why is AI $\cap$ formal math important?

Formal math for LLMs

- **Verifiable**
  - Prevent incorrect math and code generation
  - Feedback signal for learning
- Tests **reasoning**
  - From easy: 1+1 = 2
  - To hard: Fermat's Last Theorem
- Complementary **tools**
  - Computer algebra, SAT/SMT solvers, ...
  - Rule-based automation, ...

And still far from being "solved" (April 2024)...

LLMs ∩ formal math

Generative Language Modeling for Automated Theorem Proving

Stanislas Polu
OpenAI
spolu@openai.com

Ilya Sutskever
OpenAI
ilyasu@openai.com

Figure 5: *gpt-f* (2020)

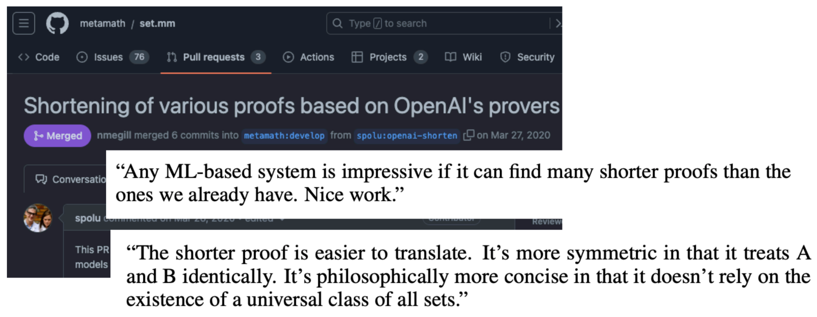

Figure 6: *gpt-f* (2020)

LLMs ∩ formal math

Terence Tao
@tao@mathstodon.xyz

Finished formalizing in #Lean4 the proof of an actual new theorem (Theorem 1.3) in my recent paper arxiv.org/abs/2310.05328 :

The ability of Github copilot to correctly anticipate multiple lines of code for various routine verifications, and inferring the direction I want to go in from clues such as the names I am giving the theorems, continues to be uncanny.

Figure 7: Terence Tao's Lean formalization project (October 2023)

This lecture

1. Intro: Foundation models ∩ mathematics
    - Informal and formal mathematics
    - Why is formal mathematics important?
2. Part I: Build a LLM formal theorem proving tool
    - Data, training, proof search, evaluation, tool
3. Part II: Leveraging *informal* mathematical data
    - Via foundation model
    - Via translation

This lecture

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Part I:
Build a [L]LM proving tool

Build a [L]LM proving tool[2]

Topic	Notebook
0. Intro	notebook
1. Data	notebook
2. Learning	notebook
3. Proof Search	notebook
4. Evaluation	notebook
5. Context	notebook
6. `LLMLean` tool	notebook

Interactive notebooks and code: github.com/cmu-l3/ntptutorial-II

[2] Update of *Tutorial on neural theorem proving*, IJCAI 2023, github.com/wellecks/ntptutorial

Build a [L]LM proving tool[3]

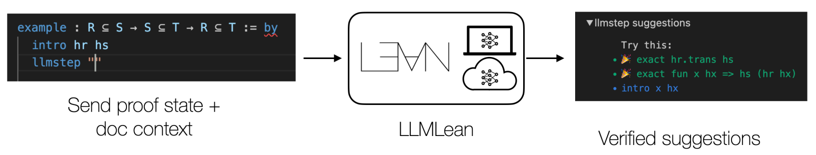
Artifacts:

Name	Huggingface
Data: mathlib extractions	l3lab/ntp-mathlib
Data: instructions (state-tactic)	l3lab/ntp-mathlib-instruct-st
Data: instructions (+context)	l3lab/ntp-mathlib-instruct-ctx
Model: state-tactic	l3lab/ntp-mathlib-st-deepseek-coder-1.3b
Model: +context	l3lab/ntp-mathlib-context-deepseek-coder-1.3b

Datasets and models on Huggingface: https://huggingface.co/l3lab

[3] Update of *Tutorial on neural theorem proving*, IJCAI 2023, github.com/wellecks/ntptutorial

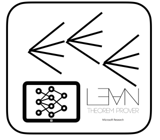
Build a [L]LM proving tool[4]



[4] Update of *Tutorial on neural theorem proving*, IJCAI 2023, github.com/wellecks/ntptutorial

Next-step (tactic) prediction

- Language model suggests next-proof-steps
- Generate a full proof via tree search



[1] E.g., [Polu & Sutskever 2020], [Han et al 2021], [Jiang et al 2022], [Yang et al 2023]

Language models

- Model: $p_\theta(\mathsf{y}|\mathsf{x};\mathcal{D})$
  - y : output sequence
  - x : input sequence
  - θ : parameters (e.g., transformer)
  - $\mathcal{D}$: dataset

Language models

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- Learning:
    - $\arg\max_\theta \sum_{\mathsf{y}\in\mathcal{D}} \log p_\theta(\mathsf{y})$

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    - θ : parameters (e.g., transformer)
    - $\mathcal{D}$: dataset
- Learning:
    - $\arg\max_\theta \sum_{\mathsf{y}\in\mathcal{D}} \log p_\theta(\mathsf{y})$
- Inference:
    - $\mathsf{y} = f(p_\theta(\cdot|x))$
    - f: e.g., sampling

Problem setup

Proof: sequence of (state, step)

- $(x_0, y_0), \ldots, (x_T, y_T)$
- x_t: proof state from Lean
- y_t: proof step ("tactic")

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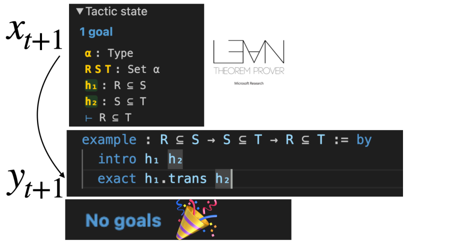

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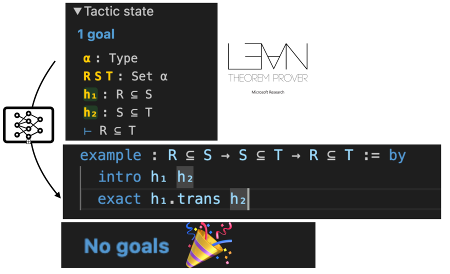
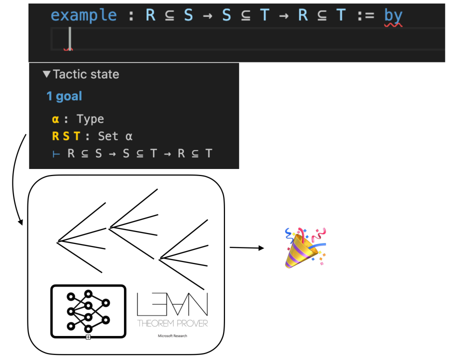


Idea: train language model $p_\theta(y_t|x_t)$ on a dataset of (state, step) pairs

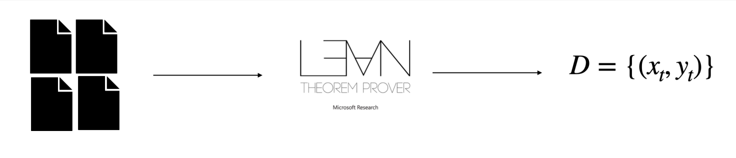
Problem setup

Proof: sequence of (state, step)

- $(x_0, y_0), \ldots, (x_T, y_T)$
- x_t: proof state from Lean
- y_t: proof step ("tactic")



...then use the model + tree search to prove full theorems



- Extract (state, tactic) pairs from Lean projects. Tools:
    - ntp-training-data (this tutorial)[5]
    - Lean Dojo [2]

[5] Based on github.com/semorrison/lean-training-data

1. Data (ntp-training-data)

- Format each (state, tactic) pair as (prompt, completion) example:

Prompt:

```
/- You are proving a theorem in Lean 4.
You are given the following information:
- The current proof state, inside [STATE]...[/STATE]

Your task is to generate the next tactic in the proof.
Put the next tactic inside [TAC]...[/TAC]
-/
[STATE]
m n : ℕ
h : Nat.Coprime m n
⊢ Nat.gcd m n = 1
[/STATE]
[TAC]
```

Completion:

```
rw [Nat.Coprime] at h
[/TAC]
```

Mathlib gives a dataset of $\approx$ 300,000 examples

1. Data (ntp-training-data)

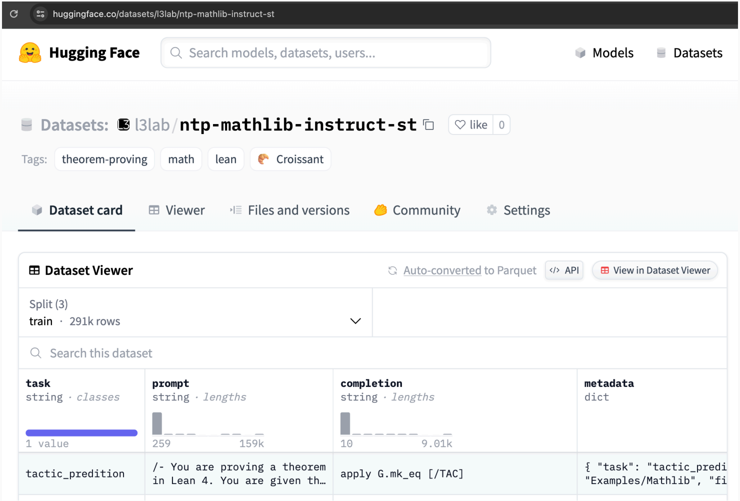


Figure 8: Data on Huggingface

[NOTEBOOK DEMO]

2. Learning

- Standard supervised learning on $\mathcal{D}$:

$$\arg\max_{\theta} \sum_{(x_t, y_t) \in \mathcal{D}} \log p_{\theta}(y_t | x_t)$$

Learning (ntp-tune)

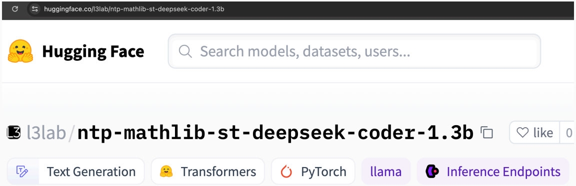
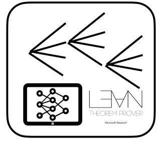


Figure 9: Trained tutorial model on Huggingface

[NOTEBOOK DEMO]

3. Proof search

- Use generator $p_\theta(y_t|x_t)$ to generate a full proof $y_1, \ldots, y_T$
- Standard approach: *Best-first search*



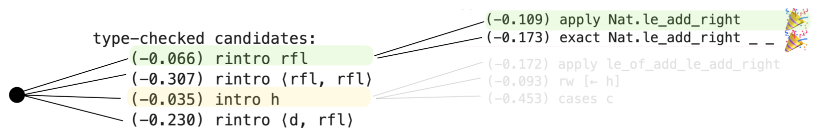
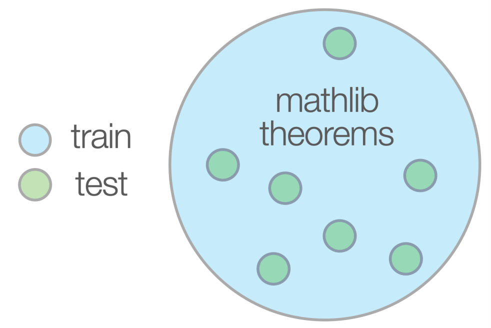


Figure 10: Best-first search[6]

[6] Example scoring function $\frac{1}{Z}\sum_t \log p_\theta(y_t|x_t)$

[NOTEBOOK DEMO]

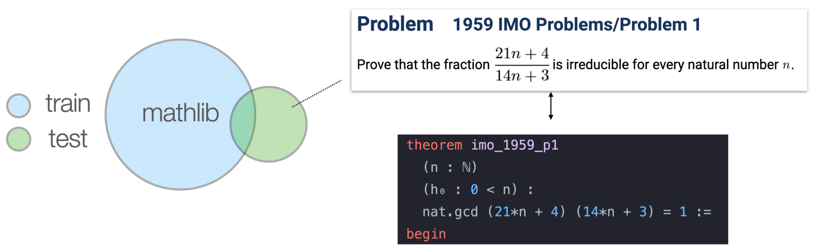
- Proof search on held-out theorems from the training distribution



4. Evaluation | benchmarks

Benchmarks evaluate problems drawn from a different distribution:

- **miniF2F** [3]: competition problems (AMC, AIME, IMO)



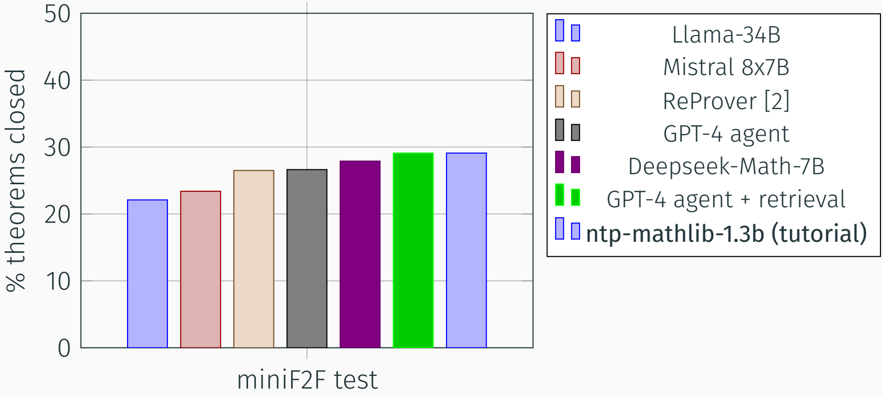


Figure 11: Proof search performance on miniF2F theorems. **The model we trained in the tutorial notebooks gets 29.1% (71/244) on miniF2F test.**

4. Evaluation

```
-- from mathlib:
theorem prod_mono
 ⦃s₁ s₂ : Subsemiring R⦄ (hs : s₁ ≤ s₂)
 ⦃t₁ t₂ : Subsemiring S⦄ (ht : t₁ ≤ t₂) :
 s₁.prod t₁ ≤ s₂.prod t₂ := by
    intro x hx
    simp_rw [Subsemiring.mem_prod]
    cases' x with x_fst x_snd
    exact ⟨hs hx.1, ht hx.2⟩

-- from miniF2F:
theorem mathd_algebra_159 (b : ℝ) (f : ℝ → ℝ)
 (h₀ : ∀ x, f x = 3*x^4 - 7*x^3 + 2*x^2 - b*x + 1)
 (h₁ : f 1 = 1) : b = -2 := by
    apply eq_neg_of_add_eq_zero_left
    rw [h₀] at h₁
    norm_num at h₁
    linarith
```

Figure 12: Generated proofs

4. Evaluation

[NOTEBOOK DEMO]

5. Extensions: context

Real theorem proving uses *context* (e.g., new definitions, lemmas) [1]:

```
variable {Ω : Type*}[Fintype Ω]

structure my_object (Ω : Type*)[Fintype Ω] :=
  (f : Ω → ℝ)
  (cool_property : ∀ x : Ω, 0 ≤ f x)

theorem my_object_sum_nonneg (o1 o2: my_object Ω) : o1.f + o2.f ≥ 0 := by
  apply add_nonneg
  . apply o1.cool_property
  . apply o2.cool_property
```

Figure 13: The theorem uses a newly defined *my_object* and *cool_property* [1].

A model trained on (state, tactic) examples does not access context outside of its training set.

5. Extensions: context

Train a context-dependent model:

$$p_\theta(y_t|x_t, c_t), \tag{1}$$

e.g., where c is the preceding file contents.

5. Extensions: context

Prompt:

```
/- You are proving a theorem in Lean 4.
You are given the following information:
- The file contents up to the current tactic, inside [CTX]...[/CTX]
- The current proof state, inside [STATE]...[/STATE]

Your task is to generate the next tactic in the proof.
Put the next tactic inside [TAC]...[/TAC]
-/
[CTX]
import Mathlib.Data.Nat.Prime

theorem test_thm (m n : Nat) (h : m.Coprime n) : m.gcd n = 1 := by

[/CTX]
[STATE]
m n : ℕ
h : Nat.Coprime m n
⊢ Nat.gcd m n = 1
[/STATE]
[TAC]
```

Completion:

```
rw [Nat.Coprime] at h
[/TAC]
```

[NOTEBOOK DEMO]

LLMLEAN: tools that integrate LLMs into the Lean proof assistant

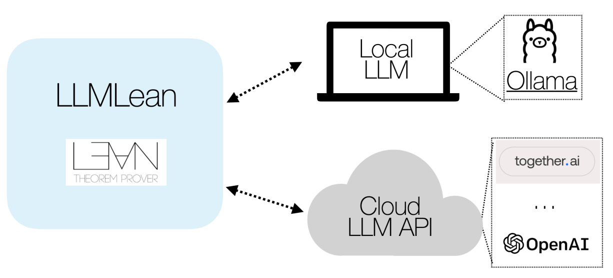


Figure 14: github.com/cmu-l3/llmlean

Example: Verified *llmstep*[7] suggestions on a MacBook Pro:

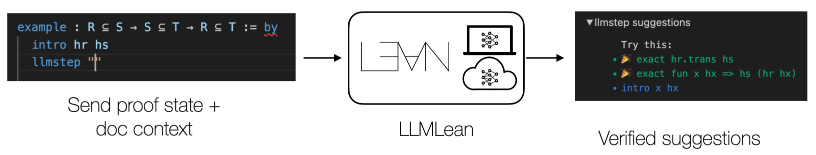


Figure 15: github.com/cmu-l3/llmlean

[DEMO]

[7] *LLMstep: LLM proofstep suggestions in Lean* [Welleck & Saha 2023]

Recap

- Train next-tactic generators, $p_\theta(y_t|x_t, c_t)$
- Prove theorems with a best-first tree search
- Data, Learning, Search, Evaluation, LLMLean tool

Recap

Check out the notebooks, code, data, and models!

- Notebooks and code: github.com/cmu-l3/ntptutorial-II
    - Data: NTP-TRAINING-DATA
    - Proof search: NTP-INTERACT
    - Fine-tuning: NTP-TUNE
- Datasets and models:
    - https://huggingface.co/l3lab

Extensions (formal-to-formal tactic generation)

Active research area with many extensions and related works:

Reinforcement learning

- Expert Iteration [Polu et al 2022]
- Thor with Expert Iteration [Wu et al 2022]

Search algorithms

- Hypertree Proof Search [Lample et al 2022]
- DT-Solver [Wang et al 2023]

Retrieval

- Reprover [Yang et al 2023]

Integrating symbolic provers

- Thor [Jiang et al 2022]

LLM agents

- COPRA [Thakur et al 2024]

Benchmarks

- miniF2F [Zheng et al 2021]
- ProofNet [Azerbayev et al 2023]

Data extraction/interaction

- PISA [Jiang et al 2021] (Isabelle)
- PACT [Han et al 2021] (Lean 3)
- lean-training-data [Morrison 2023]
- ntp-training-data (this tutorial)
- Lean Dojo [Yang et al 2023]

Tools

- LLMlean/llmstep [Welleck & Saha 2023]
- Lean Copilot [Song et al 2023]

Non-exhaustive list; also more extensions in the next section!

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    - Informal and formal mathematics
    - Why is formal mathematics important?
2. Part I: Build a LLM formal theorem proving tool
    - Data, training, proof search, evaluation, tool
3. Part II: Leveraging *informal* mathematical data
    - Via foundation model
    - Via translation and guidance

Part II: Leveraging *informal* mathematical data

Leveraging *informal* mathematical data

- *Previous*: train a model purely on formal data

Leveraging *informal* mathematical data

- *Previous*: train a model purely on formal data
- *Next*: use *informal* mathematical data
    - Latex proofs
    - Math textbooks, papers, websites, ...
    - Conversations, ...

Leveraging *informal* mathematical data

Why leverage informal data?

1. Data scarcity
    - Lean: 300 million tokens
    - Arxiv: 29 billion tokens
    - General web data: > 5 trillion tokens

Leveraging *informal* mathematical data

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2. Guiding search
    - Informal proofs can help cut down the space of possible proofs

Leveraging *informal* mathematical data

Why leverage informal data?

1. Data scarcity
   - Lean: 300 million tokens
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   - General web data: > 5 trillion tokens

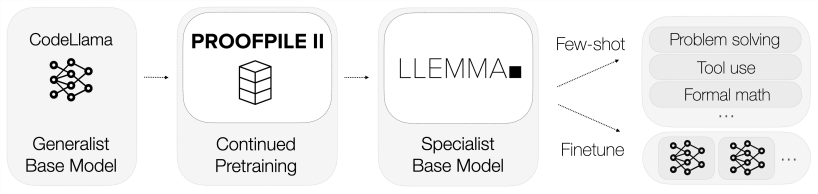
   $\rightarrow$ ***transfer knowledge by adapting a generalist model to mathematical data***

2. Guiding search
   - Informal proofs can help cut down the space of possible proofs

Foundation model for mathematics : LLEMMA[8]

Recipe for adapting a language model to mathematical data:

- Collect high-quality, diverse math-related corpus, PROOFPILE II
- Continue pretraining a base model (e.g., Code LLama) on PROOFPILE II



[8] *Llemma: an open language model for mathematics*
Zhangir Azerbayev, Hailey Schoelkopf, Keiran Paster, Marco Dos Santos, Stephen Marcus McAleer, Albert Q. Jiang, Jia Deng, Stella Biderman, Sean Welleck

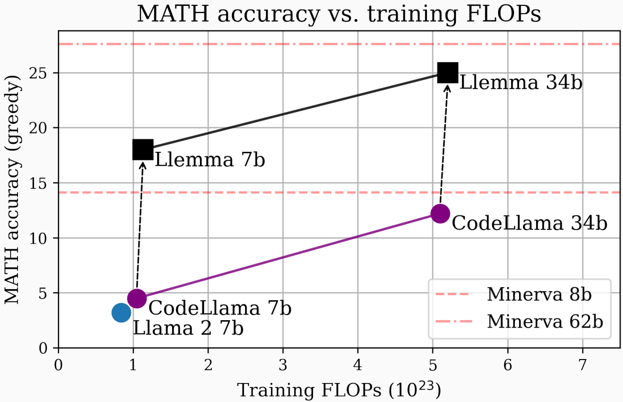


Figure 16: LLEMMA improves with a modest amount of math-specific compute

Proofpile II : code + web data + Arxiv papers

- AlgebraicStack – 11B tokens from 17 programming languages
  - 1.5B tokens of formal math
  - Extracted Lean and Isabelle goal states

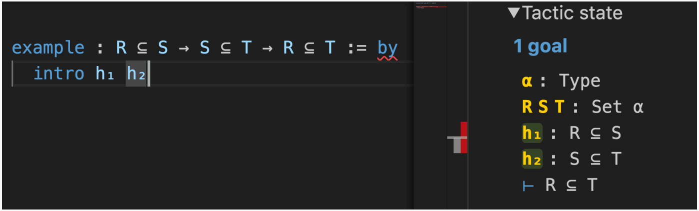


Figure 17: Lean code (left) and goal state (right)

LLEMMA | web: OPENWEBMATH [Paster et al 2023][9]

- 14.7 billion tokens of math-related web data

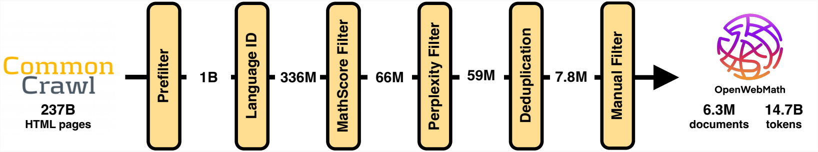


Figure 18: OpenWebMath pipeline.

[9] *OpenWebMath: An Open Dataset of High-Quality Mathematical Web Text.* Keiran Paster, Marco Dos Santos, Zhangir Azerbayev, Jimmy Ba

Llemma | formal theorem proving

Traditional proof search: p_θ(next-tactic|state) + best-first search.

- We implement a *few-shot* version by providing Llemma with 3 (*state, next-tactic*) examples in its prompt

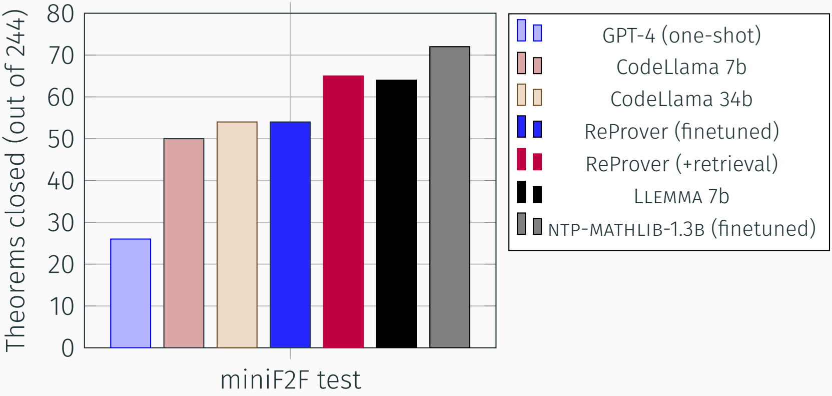
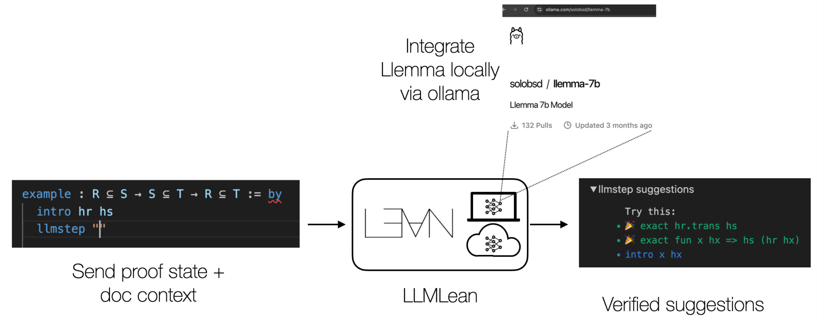


Figure 19: Few-shot proving in Lean with LLEMMA

LLEMMA on your laptop with LLMLEAN [10]



DEMO

[10] https://github.com/cmu-l3/llmlean, based on *LLMstep: LLM proofstep suggestions in Lean*. Sean Welleck & Rahul Saha, Neurips Math+AI 2023

Recap

- Leverage informal data by pretraining + adaptation to diverse mathematical data
    - Llemma: 7B and 34B CodeLLama further trained on Proofpile II
- Open platform for research:
    - Code/Models/Data: *https://github.com/EleutherAI/math-lm*

Leveraging *informal* mathematical data

Why leverage informal data?

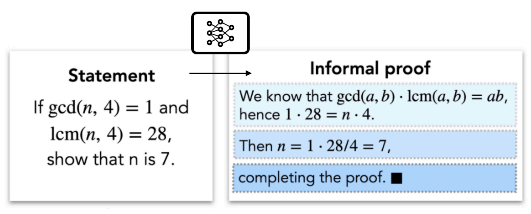
1. Data scarcity

2. Guiding search
    - Informal proofs can help cut down the space of possible proofs

Draft-sketch-prove[11]

Given informal theorem x_I,
formal theorem x_F

1. **Draft** $y_I \sim p(\cdot|x_I)$

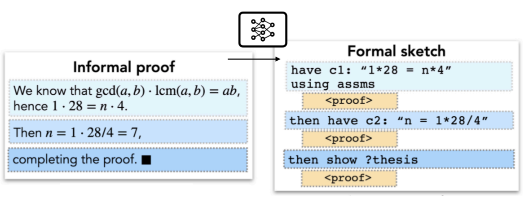


[11] *Draft, Sketch, Prove: Guiding Formal Theorem Provers with Informal Proofs*
Jiang, Welleck, Zhou, Lacroix, Liu, Li, Jamnik, Lample, Wu. ICLR 2023

Draft-sketch-prove[11]

Given informal theorem x_I,
formal theorem x_F

1. Draft $y_I \sim p(\cdot|x_I)$
2. **Sketch** $z_F \sim p(\cdot|x_F, x_I, y_I)$



[11] *Draft, Sketch, Prove: Guiding Formal Theorem Provers with Informal Proofs*
Jiang, Welleck, Zhou, Lacroix, Liu, Li, Jamnik, Lample, Wu. ICLR 2023

Draft-sketch-prove[11]

Given informal theorem x_I,
formal theorem x_F

1. Draft $y_I \sim p(\cdot|x_I)$
2. Sketch $z_F \sim p(\cdot|x_F, x_I, y_I)$
3. **Prove** $y_F = f(x_F, z_F)$

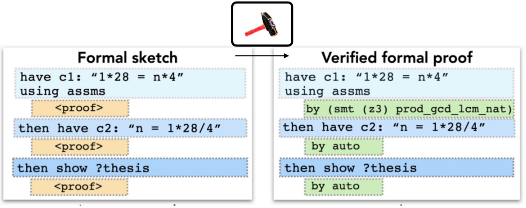


Figure 20: "Classical" prover *Sledgehammer*

[11] *Draft, Sketch, Prove: Guiding Formal Theorem Provers with Informal Proofs*
Jiang, Welleck, Zhou, Lacroix, Liu, Li, Jamnik, Lample, Wu. ICLR 2023

Draft-sketch-prove

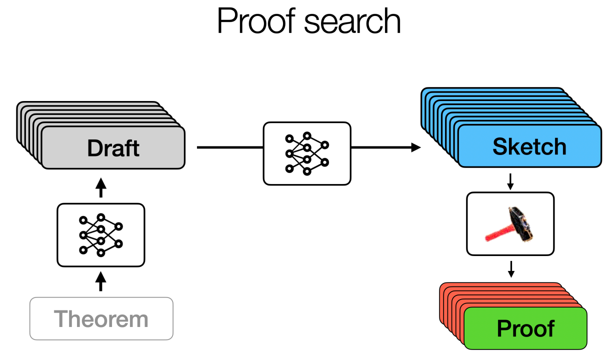


Figure 21: Proof search

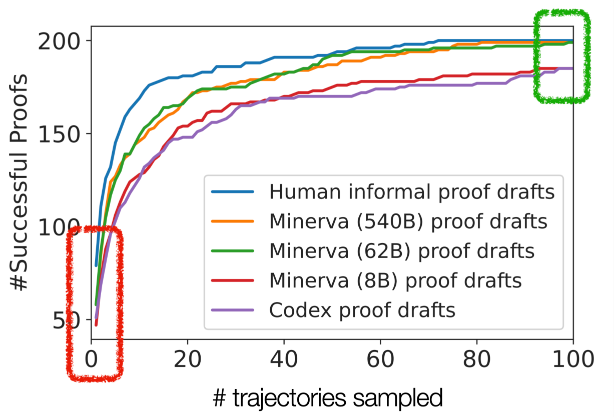


Figure 22: Proof search scaling

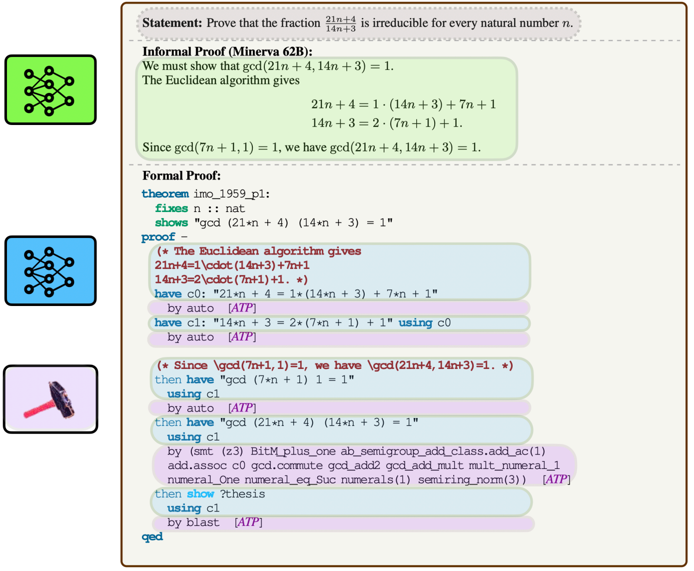


Figure 23: International Math Olympiad problem

Draft, Sketch, Prove

[DEMO NOTEBOOK]

Recap

Why leverage informal data?

1. Data scarcity
   - Adapt a foundation model to “all” mathematical data

2. Guiding search
   - Informal proofs can help cut down the space of possible proofs

Extensions[12]

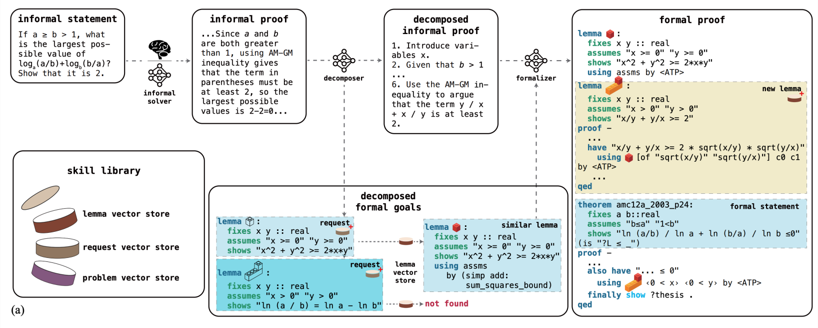


Figure 24: Lego Prover

[12] *Lego-Prover: Neural Theorem Proving with Growing Libraries*
Haiming Wang, Huajian Xin, Chuanyang Zheng, Zhengying Liu, Qingxing Cao, Yinya Huang, Jing Xiong, Han Shi, Enze Xie, Jian Yin, Zhenguo Li, Xiaodan Liang, ICLR 2024 (Oral)

Extensions[12]

Success rate	LLM	miniF2F-valid	miniF2F-test
Baselines			
Thor (Jiang et al., 2022a)	-	28.3%	29.9%
Thor + expert iteration (Wu et al., 2022)	Codex	37.3%	35.2%
Draft, sketch, and Prove (Jiang et al., 2022b)	Codex	42.6%	39.3%
Subgoal-Learning (Zhao et al., 2023)	ChatGPT	48.0%	45.5%
Ours (100 attempts)			
LEGO-Prover (model informal proof)	ChatGPT	52.0%	45.5%
LEGO-Prover (human informal proof)	ChatGPT	55.3%	**50.0%**
LEGO-Prover*	ChatGPT	**57.0%**	**50.0%**

Figure 25: Lego Prover results

[12] *Lego-Prover: Neural Theorem Proving with Growing Libraries*
Haiming Wang, Huajian Xin, Chuanyang Zheng, Zhengying Liu, Qingxing Cao, Yinya Huang, Jing Xiong, Han Shi, Enze Xie, Jian Yin, Zhenguo Li, Xiaodan Liang, ICLR 2024 (Oral)

Extensions and related (leveraging informal data)

Active research area with many extensions and related works:

Math foundation models

- Minerva [Lewkowycz et al 2022] (*inaccessible*)
- Llemma [Azerbayev et al 2023]
- InternLM-Math [Ying et al 2024]
- DeepSeek-Math [Shao et al 2024]

Informal-to-formal translation and guidance

- Statements [Wu et al 2022]
- Verifying informal (DTV) [Zhou et al 2024]
- LLM Agent (COPRA) [Thakur et al 2024]
- LegoProver [Wang et al 2024]

Tools/case studies

- Codex autoformalization [Agrawal 2022]

Informal+formal benchmarks

- miniF2F+informal [Jiang et al 2023]
- ProofNet [Azerbayev et al 2023]
- MUSTARD [Huang et al 2024]

Data

- Naturalproofs-gen [Welleck et al 2022]
- MMA [Jiang et al 2024]
- OpenWebMath [Paster et al 2023]
- Proofpile-II [Azerbayev et al 2023]

Other tutorials

- Neural theorem proving, IJCAI 2023 github.com/wellecks/ntptutorial
- ML for Theorem Proving, Neurips 2023 machine-learning-for-theorem-proving.github.io

Non-exhaustive list!

Recap

1. Intro: Foundation models $\cap$ mathematics
   - Informal and formal mathematics
   - Why is formal mathematics important?
2. Part I: Build a LLM formal theorem proving tool
   - Data, training, proof search, evaluation, tool
3. Part II: Leveraging *informal* mathematical data
   - Via foundation model
   - Via translation and guidance

Thank you!

Collaborators on works in this tutorial (alphabetical by last name):

- Zhangir Azerbayev (Princeton)
- Stella Biderman (Eleuther)
- Jia Deng (Princeton)
- Marco Dos Santos (Cambridge)
- Jiewen Hu (CMU)
- Mateja Jamnik (Cambridge)
- Albert Jiang (Cambridge, Mistral)
- Timothee Lacroix (Mistral)
- Guillaume Lample (Mistral)
- Wenda Li (Edinburgh)
- Jiacheng Liu (Washington)
- Stephen McAleer (CMU)
- Keiran Paster (Toronto)
- Rahul Saha (Independent)
- Hailey Schoelkopf (Eleuther)
- Yuhuai (Tony) Wu (X.ai)
- Jin Zhou (Cornell)

https://github.com/cmu-l3/ntptutorial-II

https://huggingface.co/l3lab

Learning, Language, and Logic (L3) Lab

S. Welleck and R. Saha.
Llmstep: Llm proofstep suggestions in lean.
ArXiv, abs/2310.18457, 2023.

K. Yang, A. Swope, A. Gu, R. Chalamala, P. Song, S. Yu, S. Godil, R. Prenger, and A. Anandkumar.
LeanDojo: Theorem proving with retrieval-augmented language models.
In *Neural Information Processing Systems (NeurIPS)*, 2023.

K. Zheng, J. M. Han, and S. Polu.
minif2f: a cross-system benchmark for formal olympiad-level mathematics.
In *International Conference on Learning Representations*, 2022.