Foundations: Pretraining and scaling laws

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Part I: Foundations

- Learning
- Evaluation
- Inference
- Data
Language model learning pipeline

- Pretraining
  - Gives a “foundation model”
- Adaptation
  - Continued pretraining
  - Fine-tuning
  - Learning from feedback
  - In-context learning / prompting
Example: CodeLlama [6]

- Pretraining
  - 2 trillion (T) tokens of mixed data (web, code, etc.)
Example: CodeLlama [6]

- **Pretraining**
  - 2 trillion (T) tokens of mixed data (web, code, etc.)

- **Adaptation**
  - Continued pretraining
    - 500 billion (B) tokens of mostly code data
  - Finetuning
    - Long sequences, Python code, and/or instructions
• Recap of language models and pretraining objective
• Scaling laws for understanding pretraining
• What do these scaling laws not capture?
A language model is a probability distribution over sequences:

\[ p_{\theta}(y) \]  

- \( y = (y_1, \ldots, y_T) \)
- \( \theta \): parameters
Recap: Autoregressive neural language models

Typical language models are autoregressive, and are parameterized by a transformer:

\[ p_\theta(y) = \prod_{t=1}^{T} p_\theta(y_t|y_{<t}) \]  \hspace{1cm} (2)

- \( \theta \): transformer\(^1\)

\(^1\)For a review of transformers, see Chapter 12 of Bishop, *Deep Learning* 
https://www.bishopbook.com/.
Recap: Autoregressive neural language models

Autoregressive distributions allow for easy sampling:

- \( \hat{y}_1 \sim p_\theta(\emptyset) \)
- \( \hat{y}_2 \sim p_\theta(\cdot|\hat{y}_1) \)
- \( \ldots \)
- \( \rightarrow \hat{y} \sim p_\theta(y) \)
Autoregressive distributions allow for easy sampling:

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- ...
- $\rightarrow \hat{y} \sim p_\theta(y)$

Next: how do we learn the parameters $\theta$?
Learning: maximum likelihood

Make observed data likely under the model; *maximum likelihood*:

\[
\arg\max_{\theta} \frac{1}{|\mathcal{D}|} \sum_{y \in \mathcal{D}} \log p_\theta(y) \tag{3}
\]
Make observed data likely under the model; *maximum likelihood*:

$$\arg\max_\theta \frac{1}{|\mathcal{D}|} \sum_{y \in \mathcal{D}} \log p_\theta(y)$$  \hspace{1cm} (3)$$

- Example: $\mathcal{D}$ is 2 trillion tokens for Llama 2
Learning: next-token

Equivalently, learn to ‘predict the next token’:

\[
\begin{align*}
\arg\max_\theta & \frac{1}{|D|} \sum_{y \in D} \log p_\theta(y) \\
\equiv & \arg\min_\theta \frac{1}{|D|} \sum_{y \in D} \sum_{t=1}^T \left( - \log p_\theta(y_t|y_{<t}) \right)
\end{align*}
\]
Equivalently, match a target distribution:

$$\arg \min_{\theta} \text{KL}(q \| p_{\theta}),$$

(6)

where the dataset $\mathcal{D} \sim q$ is sampled from a target distribution $q$.\(^2\)

\(^2\text{KL: Kullback-Leibler divergence}\)
Equivalently, match a target distribution:

\[
\min_\theta KL(q \parallel p_\theta) = \min_\theta - \sum_{y \in Y} q(y) \log \frac{p_\theta(y)}{q(y)} \\
\equiv \min_\theta - \sum_{y \in Y} q(y) \log p_\theta(y) + \text{constant} \\
\equiv \min_\theta - \mathbb{E}_{y \sim q} \log p_\theta(y) \\
\approx \min_\theta - \frac{1}{|D|} \sum_{y \in D} \log p_\theta(y) \\
\equiv \max_\theta \sum_{y \in D} \log p_\theta(y) \quad \text{(Maximum likelihood!)}
\]
Next-token prediction has a nice interpretation: it fits the language model $p_\theta$ to a target distribution $q$ represented by the dataset $\mathcal{D}$. 
We want to fit the distribution better by “adding more compute”:

- “The biggest lesson that can be read from 70 years of AI research is that general methods that leverage computation are ultimately the most effective, and by a large margin”\(^3\)

\(^3\)The Bitter Lesson, Richard Sutton 2019
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What is “compute”?

\(^3\)The Bitter Lesson, Richard Sutton 2019
We spend **compute** by performing forward and backward passes using our model on token sequences.
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A rough approximation for transformer language models is [4]:

\[ C \approx 6ND \]  \hspace{1cm} (7)

- \( N \): number of model parameters
- \( D \): number of tokens
- \( C \): compute; floating point operations (FLOPs)
We spend **compute** by performing forward and backward passes using our model on token sequences.

For example, LLama 2:

\[ C \approx 6 \times 7 \text{ billion} \times 2 \text{ trillion} \]

\[ = 8.4 \times 10^{22} \text{ FLOPs} \]
We spend **compute** by performing forward and backward passes using our model on token sequences.

For example, LLama 2:

\[
C \approx 6 \times 7 \text{ billion} \times 2 \text{ trillion} \quad (8)
\]
\[
= 8.4 \times 10^{22} \text{ FLOPs} \quad (9)
\]

We can **increase compute** by increasing the **number of parameters** (\(\uparrow N\)), training on **more tokens** (\(\uparrow D\)), or a **combination** thereof.
Good news: cross entropy loss gets better with more compute [Kaplan et al 2020 [4]].

Test loss predictably improves with more compute [Kaplan et al 2020 [4]].
Good news: cross entropy loss gets better with more compute

Specifically, loss scales as a power-law with the amount of compute:

\[ L(X) \propto \frac{1}{X^{\alpha_X}}, \]  

scaling law

where \( X \) is compute \( C \), dataset size \( D \), or parameters \( N \).
Good news: cross entropy loss gets better with more compute

Example:

\[ L(C) \propto \frac{1}{C^{0.05}} \]  

(11)
Good news: cross entropy loss gets better with more compute

Basic idea:

- Train models of size $N_1, \ldots, N_n$ for $D_1, \ldots, D_d$ tokens.
- Plot loss at each step (light blue lines)
- Pick the minimum loss at each amount of compute (black line)
- Run linear regression on the resulting $(\log L, \log C)$ pairs
Figure 1: Llama training loss

Figure 2: Llama task performance
Good news: it appears to hold for code

Figure 3: Codex test loss scaling in number of parameters $N$
Good news: it appears to hold for code

Figure 4: Codex pass rate on HumanEval as a function of parameters $N$
Recap

- Pretraining is equivalent to fitting a target distribution
- The fit predictably gets better as we increase compute, as described by a scaling law
Recap

- Pretraining is equivalent to fitting a target distribution
- The fit predictably gets better as we increase compute, as described by a scaling law

Should I spend my compute on a larger model, or on more data?
Allocation:
For compute budget $C$, choose number of parameters $N$ and tokens $D$ that minimizes loss.

Investigated in "the Chinchilla paper" [Hoffmann et al 2022 [3]].
Allocation:

For compute budget $C$, choose number of parameters $N$ and tokens $D$ that minimizes loss.

$$\arg \min_{N,D} L(N, D)$$

subject to $6ND \leq C$

Investigated in “the Chinchilla paper” [Hoffmann et al 2022 [3]]
Figure 5: Previous models (e.g. Gopher) allocate a large portion of compute to model size. Chinchilla is a smaller model trained on more tokens that outperforms Gopher.
Figure 5 | **Pile Evaluation.** For the different evaluation sets in The Pile (Gao et al., 2020), we show the bits-per-byte (bpb) improvement (decrease) of *Chinchilla* compared to *Gopher*. On all subsets, *Chinchilla* outperforms *Gopher*. 

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**Allocation: Chinchilla**
To choose Chinchilla’s allocation, the authors fit scaling laws on runs with smaller amounts of compute. They used three approaches.

<table>
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<tr>
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<tbody>
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<td>1. Minimum over training curves</td>
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<tr>
<td>2. IsoFLOP profiles</td>
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<td>0.51 (0.483, 0.529)</td>
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$a \approx b$ : parameters and tokens should be scaled at the same rate.
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$a \approx b$ : parameters and tokens should be scaled at the same rate.

To understand this kind of analysis, we will look at Approach 1
Approach 1: fix $N$ and vary $D$

- For each size $N$, train 4 models with different number of tokens $D$
- For each compute $C$, pick the model with the lowest loss $L$
- We now have $(C, N, D, L)$ examples (grey points)
Approach 1: fix $N$ and vary $D$

- Fit power laws using the $(C, N, D, L)$ examples.
  - Middle: $N_{\text{opt}} \propto C^a$ (optimal model size)
  - Right $D_{\text{opt}} \propto C^b$ (optimal number of tokens)
As a recap, the slope of the lines appears in the table: scale parameters and tokens at similar rates.

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Post-Chinchilla

- The Chinchilla scaling law arguably led to a focus on scaling data.
- Trend: train on **even more tokens** than suggested by the compute-optimal scaling law.\(^4\)

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\(^4\)Training a smaller model on more tokens may be compute optimal when *inference-time compute* is factored in; smaller models require less inference compute.
Figure 6: Example: Llama 2 – more tokens than Chinchilla, equal size (70B)
DeepSeek LLM
Scaling Open-Source Language Models with Longtermism

Xiao Bi, Deli Chen, Guanting Chen, Shanhuang Chen, Damai Dai, Chengqi Deng, Honghui Ding, Kai Dong, Qiushi Du, Zhe Fu, Huazuo Gao, Kaige Gao, Wenjun Gao, Ruiqi Ge, Kang Guan, Daya Guo, Jianzhong Guo, Guangbo Hao, Zhewen Hao, Ying He, Wenjie Hu, Panpan Huang, Erhang Li, Guowei Li, Jiashi Li, Yao Li, Y.K. Li, Wenfeng Liang, Fangyun Lin, A.X. Liu, Bo Liu, Wen Liu, Xiaodong Liu, Xin Liu, Yiyuan Liu, Haoyu Lu, Shanghao Lu, Fuli Luo, Shirong Ma, Xiaotao Nie, Tian Pei, Yishi Piao, Junjie Qiu, Hui Qu, Tongzheng Ren, Zehui Ren, Chong Ruan, Zhangli Sha, Zhihong Shao, Junxiao Song, Xuecheng Su, Jingxiang Sun, Yaofeng Sun, Minghui Tang, Bingxuan Wang, Peiyi Wang, Shiyu Wang, Yaohui Wang, Yongji Wang, Tong Wu, Y. Wu, Xin Xie, Zhenda Xie, Ziwei Xie, Yiliang Xiong, Hanwei Xu, R.X. Xu, Yanhong Xu, Dejian Yang, Yuxiang You, Shuiping Yu, Xingkai Yu, B. Zhang, Haowei Zhang, Lecong Zhang, Liyue Zhang, Mingchuan Zhang, Minghua Zhang, Wentao Zhang, Yichao Zhang, Chenggang Zhao, Yao Zhao, Shangyan Zhou, Shunfeng Zhou, Qihao Zhu, Yuheng Zou

*DeepSeek-AI
Figure 7: Scaling laws for batch size and learning rate
Scaling laws as a tool in the toolbox

**Figure 8:** Predicting performance of larger models
Recap

• Scaling laws can determine “compute-optimal training”
  • I.e., the choice of $N$ and $D$ that minimizes loss at compute budget $C$.
• Scaling the amount of data is important!!
What if we run out of data?
Data-constrained scaling

Data-constrained setting

- We might want to train on much more than 2 trillion tokens
- Some programming languages have less tokens
  - E.g. Starcoder pretraining data: $\approx 300$ billion code tokens
  - E.g. Lean has $\approx 300$ million tokens [1]
Data-constrained scaling

Option 1: repeat the data

• Studied in *Scaling Data-Constrained Language Models* [5]
Data-constrained scaling

**Finding:** repeating can be good

- 4 epochs is nearly as good as 1 epoch with 4x the data
Option 2: mix in other data

- $N_1$ web tokens + $N_2$ code tokens $\approx$ repeating $N_1$ web tokens
Data-constrained scaling

Option 3: transfer

- Pretrain on $\mathcal{D} \sim q$ (e.g. web)
- Continue training on $\mathcal{D}' \sim q'$ (e.g. code)
Effective data transfer: code tokens saved by pretraining on text
Scaling laws of transfer

![Graphs showing the scaling laws for transfer](image)

**Figure 10:** Scaling Laws for Transfer [2]

Low-data setting: without pretraining on text, we get no benefit from increasing parameters.
Data-constrained scaling: Llemma

LEMMA [1]:

- Pretrain on web and code
  - Initialize with $\theta_{\text{codellama}}$
- Transfer to specialized programming languages and math
  - Continue training on $\mathcal{D}'$ : 55 billion token PROOFPILE II
Data-constrained scaling: Llemma

Llemma [1]:

- Pretrain on web and code
  - Initialize with $\theta_{\text{codellama}}$
- Transfer to specialized programming languages and math
  - Continue training on $\mathcal{D}'$: 55 billion token PROOFPILE II
    - Mathematical code (e.g., Lean)
    - Mathematical web data
    - Scientific papers
Figure 11: LLEMMMA improves with a modest amount of math-specific compute
Recap

To keep reducing loss, we need many tokens. What if we run out?

- Repeating tokens can be a useful allocation of compute
- Leverage tokens from a data-rich distribution (e.g. web text)
Summary

- Pretraining fits the distribution of pretraining data
- Scaling laws let us forecast performance, allocate compute, and choose hyperparameters
- In low-data settings: repeat data, mix in other data, transfer
Looking ahead

What do these scaling laws not cover?

• Data quality: 'better' data may be more compute efficient
• Training objective: next-token may not be optimally efficient
• Distribution mismatch: what if we perfectly fit $q$, but want $q'$
  • $q$: code on the internet
  • $q'$: code that satisfies a user's intent
• Many others: architecture, inference cost, performance metric,...

We will discuss all of these during the semester!
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**Llemma: An open language model for mathematics.** 

**Scaling laws for transfer, 2021.**

**Training Compute-Optimal Large Language Models.** 


ArXiv, abs/2308.12950, 2023.
Appendix
Step 1: hypothesize a scaling law

\[ L(N, D) = E + \frac{A}{N^\alpha} + \frac{B}{D^\beta} \]  \hspace{1cm} (12)
Step 1: hypothesize a scaling law

\[ L(N, D) = E + \frac{A}{N^\alpha} + \frac{B}{D^\beta} \]  

(13)

“Entropy term”: with infinite parameters and infinite data \((N, D \to \infty)\), we should approach the minimum achievable loss (entropy).
Step 1: hypothesize a scaling law

\[ L(N, D) = E + \frac{A}{N^\alpha} + \frac{B}{D^\beta} \]  \hspace{1cm} (14)

“Modeling cost”: with infinite data \((D \to \infty)\), we should incur a cost from using a transformer with \(N\) parameters.
Step 1: hypothesize a scaling law

\[ L(N, D) = E + \frac{A}{N^\alpha} + \frac{B}{D^\beta} \quad (15) \]

“Optimization cost”: with infinite parameters \((N \to \infty)\), we should incur a cost from using only \(D\) tokens with gradient descent.
Step 2: fit constants $E, A, \alpha, B, \beta$ using losses from training runs

$$L(N, D) = E + \frac{A}{N^{0.34}} + \frac{B}{D^{0.28}}$$  (16)
Step 3: derive the optimal parameters and tokens from $L$, plug in $\alpha, \beta$:

\[ N_{opt}(C) = G \left( \frac{C}{6} \right)^{a}, \quad D_{opt}(C) = G^{-1} \left( \frac{C}{6} \right)^{b}, \quad \text{where} \quad G = \left( \frac{\alpha A}{\beta B} \right)^{\frac{1}{\alpha + \beta}}, \quad a = \frac{\beta}{\alpha + \beta}, \quad \text{and} \quad b = \frac{\alpha}{\alpha + \beta} \]

Result: $a = 0.46, b = 0.54$